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etc., etc. Whence we deduce, $a=2^{n-1}[r-(n-1)s]$; $b=2^{n-2}\{r-[n-(2^2-1)]s\}$; $c=2^{n-3}\{r-[n-(2^3-1)]s\}$; $d=2^{n-4}\{r-[n-(2^4-1)]s\}$; etc., etc. But in order that all values are positive, we have the condition $r>(n-1)s$. Therefore, put $r=(n-1)s+t$, and we obtain,

$$\begin{array}{ll}
a=2^{n-1}t, & x_1=2^{n-1}[s(n-2)+t]+s, \\
b=2^{n-2}(2s+t), & x_2=2^{n-2}[s(n-2)+t]+s, \\
c=2^{n-3}2(2^2-1)s+t, & x_3=2^{n-3}[s(n-2)+t]+s, \\
d=2^{n-4}2(2^3-1)s+t, \text{ etc.} & x_4=2^{n-4}[s(n-2)+t]+s, \text{ etc.}
\end{array}$$

$s$ and $t$ may be any values, integral or fractional; but $s$ may also be *zero*. When $s=0$, the money each had at first equals, respectively, what he had at the end of the transaction; or $a=x_1=2^{n-1}t$; $b=x_2=2^{n-1}t$; ...... $x_{n-1}=2^2t$; $x_n=2t$. When $t=2s$, all have an equal sum at the end of the transaction; or $2^ns=a=b=c$, etc. For obtaining integral values this form of solution is preferable.

**IV. Solution by F. M. McGAW, A. M., Professor of Mathematics, Bordentown Military Institute, Bordentown, N. J.**

This problem is concerned with a double $n$-fold series. Let $A$, $B$, $C$, etc., represent the amounts which each person has to start with; then after the several exchanges, we have: $A$'s $n$th term$=2^{n-1}[2A-\Sigma(A)]=a$; where $\Sigma(A)$ means $(A+B+C+\ldots\ldots$ to $n$ letters); $B$'s $n$th term equals $2^{n-2}[4B-\Sigma(A)]=b$; $C$'s $n$th term$=2^{n-3}[8C-\Sigma(A)]=c$; and so on for $R$'s $n$th term$=2^{n-r}[2^rR-\Sigma(A)]=\rho$; and $N$'s $n$th term$=2^{n-n}[2^nN-\Sigma(A)]=\nu$.

Expanding, we may write,

$$\left.\begin{array}{r}
(2^n-2^{n-1})A-2^{n-1}B-2^{n-1}C\ldots\ldots-2^{n-1}R\ldots\ldots-2^{n-1}N=a \\
-2^{n-2}A+(2^n-2^{n-2})B-2^{n-2}C\ldots\ldots-2^{n-2}R\ldots\ldots-2^{n-2}N=b \\
-2^{n-3}A-2^{n-3}B+(2^n-2^{n-3})C\ldots\ldots-2^{n-3}R\ldots\ldots-2^{n-3}N=c \\
\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
-2^{n-r}A-2^{n-r}B-2^{n-r}C\ldots\ldots+(2^n-2^{n-r})R\ldots\ldots-2^{n-r}N=\rho \\
\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
-2^{n-n}A-2^{n-n}B-2^{n-n}C\ldots\ldots-2^{n-n}R\ldots\ldots+(2^n-2^{n-n})N=\nu
\end{array}\right\}\text{I.}$$

Whence by determinants,

$$\Delta.A=\left|\begin{array}{lllll}
a, & -2^{n-1}, & -2^{n-1}, & \ldots-2^{n-1}, & \ldots-2^{n-1} \\
b, & 2^n-2^{n-2}, & -2^{n-2}, & \ldots-2^{n-2}, & \ldots-2^{n-2} \\
c, & -2^{n-3}, & 2^n-2^{n-3}, & \ldots-2^{n-3}, & \ldots-2^{n-3} \\
\ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \\
\rho, & -2^{n-r}, & -2^{n-r}, & \ldots 2^n-2^{n-r}, & \ldots-2^{n-r} \\
\ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \\
\nu, & -1, & -1, & \ldots-1, & \ldots 2^n-1
\end{array}\right|$$

Where $\Delta$ is the determinant of the coefficients.

Also solved by *H. C. WILKES.*

**80. Proposed by G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics, The Russell College, Lebanon, Va.**

Solve $1+x^4=a(1+x)^4$.

I. **Solution by COOPER D. SCHMITT, A. M., Professor of Mathematics, University of Tennessee, Knoxville, Tenn.; P. S. BERG, A. M., Principal of Schools, Larimore, N. D.; J. OWEN MAHONEY, B. E., M. Sc., Graduate Fellow in Mathematics in Vanderbilt University, Lynnville, Tenn.; CHAS. A. JONES, Torrance, Miss.; and the PROPOSER.**

$1+x^4=a+4ax+6ax^2+4ax^3+ax^4.$

$x^2+(1/x^2)=(a/x^2)+(4a/x)+6a+4ax+ax^2.$

$[x+(1/x)]^2=a[x+(1/x)]^2+4a[x+(1/x)]+4a+2\ldots\ldots(1).$

Let $x+(1/x)=y\ldots\ldots(2)$; $\therefore\ x^2-xy=1.$

$\therefore\ x=\frac{1}{2}[y\pm\sqrt{(y^2-4)}]\ldots\ldots(3).$ (2) in (1) gives

$y^2+[4a/(a-1)]y=-[(4a+2)/(a-1)].$

$\therefore\ y=[\pm\sqrt{(2a+2)}-2a]/(a-1)\ldots\ldots(4).$ (4) in (3) gives

$x=[1/2(a-1)]\{-2a\pm\sqrt{[2a+2]}\pm\sqrt{[10a-2\pm4a\sqrt{(2a+2)}]}\}.$

II. **Solution by J. SCHEFFER, A. M., Hagerstown, Md.; and F. M. McGAW, A. M., Professor of Mathemetics in Bordentown Military Institute, Bordentown, N. J.**

Expanding and arranging we get, dividing by $a-1$,

$$x^4+\frac{4a}{a-1}x^3+\frac{6a}{a-1}x^2+\frac{4a}{a-1}x+1=0,$$

a reciprocal equation which is easily reduced by dividing by $x^2$, thus

$$[x^2+(1/x^2)]+\frac{4a}{a-1}[x+(1/x)]+\frac{6a}{a-1}=0,$$

Putting $x+(1/x)=y$, we shall have

$$y^2+\frac{4a}{a-1}y=-\frac{4a+2}{a-1},\ \text{whence}\ y=-\frac{2a}{a-1}\pm\frac{1}{a-1}\sqrt{2(a+1)},$$

and then

$$x=-\frac{a}{a-1}\pm\frac{1}{2(a--1)}\sqrt{2(a+1)}\pm\frac{1}{2(a-1)}\sqrt{2(2a^2+a+1)\mp4a\sqrt{2(a+1)}}.$$

**Also solved by *A. H. BELL.***

## GEOMETRY.

84. **Proposed by FREDERICK R. HONEY, Ph. B., Instructor in Mathematics in Trinity College, New Haven, Conn.**

Find the locus of a point which will trisect all arcs having a common chord.

I. **Solution by G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics, The Russell College, Lebanon, Va.**

In what follows the arc of the circle is assumed.

Let $O$ be the origin, $EO=z$, $AB=2c$, $AC=CD=DB=n$.

Then $BF=\frac{1}{2}(2c-n)$, $OF=\frac{1}{2}n$, $DF=\sqrt{n^2-\frac{1}{4}(2c-n)^2}$, $EB^2=c^2+z^2$, $ED^2=EB^2=c^2+z^2$, $EG^2=c^2+z^2-\frac{1}{4}n^2$.